

# Equation of State and Transport Coefficients of Relativistic Nuclear Matter

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## Abstract.

In order to evaluate qualitatively the space-time evolution of hot and dense nuclear matter the underlying equation of state and transport coefficients must be known. In this study a specific equation of state is studied: the pion gas. The classical or standard transport coefficients, namely the bulk viscosity, shear viscosity and thermal conductivity are divided by the relaxation times for the corresponding dissipative fluxes and then studied as a function of mass to temperature ratio.

## 1. Introduction

In the heavy-ion experiments such as those at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) a hot and dense matter is created. The dynamical evolution of such matter proceeds through stages consisting of both sub-hadronic (quarks and gluons) and hadronic (mesons and baryons) degrees of freedom. A knowledge of the equation of state and transport coefficients or relaxation times of various dissipative processes is essential for a complete description. For the matter produced at RHIC and LHC one would like to extract the equation of state and the transport coefficients (and/or the associated time/length scales) for a given model of the interacting matter. Then one can study the sensitivity of the space-time evolution of the system and the calculated distributions of the hadrons to the equation of state and to the dissipative, non-equilibrium processes. In the end one compare the predicted distributions with those observed in experiments.

The hot and dense relativistic nuclear matter can appear in the form of hadronic matter or quark matter. While the hot matter might have existed in the early universe the dense matter is supposed to be found in the interior of neutron stars. The hot and dense nuclear matter is created in the relativistic heavy ion collisions such as those at RHIC and those at LHC. When the space-time evolution of hot and dense matter is described by dissipative, non-equilibrium fluid dynamics as is done in [1, 2, 3, 4, 5, 6] knowledge of the equation of state and transport coefficients is essential.

While the equation of state of the relativistic nuclear matter relates the state variables the transport coefficients determine the strength of deviations from equilibrium. The transport coefficients govern the dynamics of relaxation of the matter towards equilibrium state.

Transport processes play an important role in the burning of neutron star into a strange strange quark matter as described in [7]. When a neutron at the phase boundary enters the quark matter, the quarks become deconfined into a  $u$  and two  $d$  quarks. The  $d$  quarks are transported into the burning region where they are converted to  $s$  quarks. The  $s$  quarks are then transported to the phase boundary or the burning front. The  $d$  and  $s$  quarks diffuse through each other and the  $u$  quarks. This generate flows of different quark flavors relative to each other. The burning drives the diffusion which is balanced by friction due to collisions. In the hadronic mixtures as discussed in [8] transport properties plays an important role in determining the relaxation times for different components. The viscosities and thermal conductivities in fluid mixtures, e.g., pion-kaon  $\pi K$  or quark-gluon  $qg$  mixture, should be larger than those in single component fluids, e.g.,  $\pi\pi$ ,  $qq$  or  $gg$ . In the mixture the diffusion coefficients depend not only on the total density but also on the concentrations.

## 2. The equation of state of hot and dense nuclear matter

The equation of state of relativistic nuclear matter is not that well known. The equation of state for hadronic matter is often approximated by an equation of state of free resonance gas. The hadronic equation of state is also affected by the in-medium modifications. The equation of state of quark matter is obtained from lattice QCD.

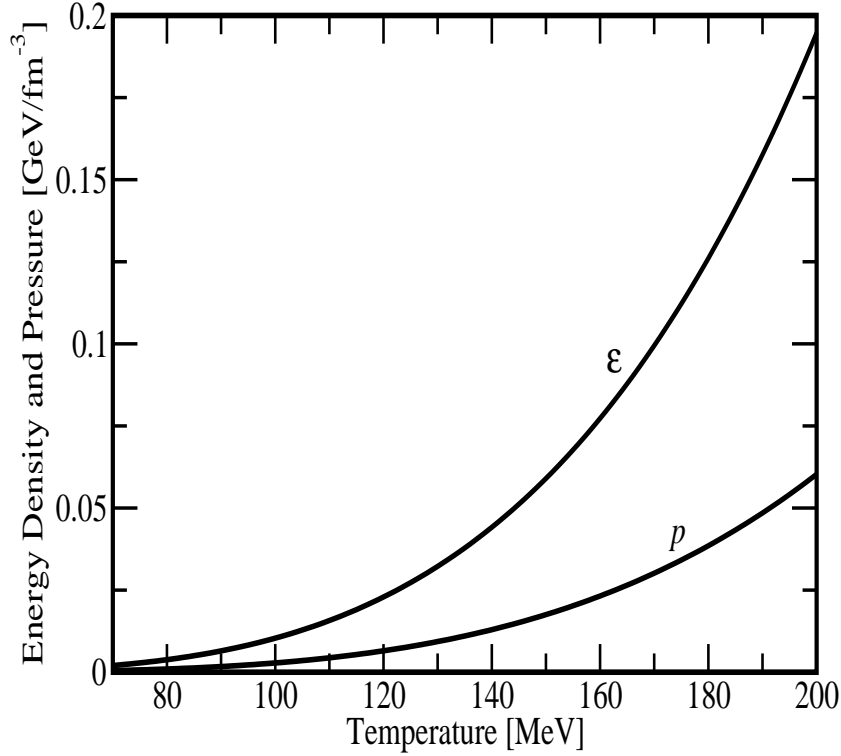
In this work the equation of state is taken to be that of resonance gas of pions only. The energy density, number density and pressure are respectively given by

$$\varepsilon(T, \mu) = \sum_k g_k \int \frac{d^3p}{(2\pi)^3} \frac{E_k}{e^{\frac{E_k - \mu}{T}} - 1}, \quad (1)$$

$$n(T, \mu) = \sum_k g_k \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{\frac{E_k - \mu}{T}} - 1}, \quad (2)$$

$$p(T, \mu) = \sum_k g_k \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_k} \frac{1}{e^{\frac{E_k - \mu}{T}} - 1}, \quad (3)$$

where  $g_k$  is a degeneracy factor. In these calculations the pion chemical potential  $\mu$  is fixed to zero. In Fig. 1 we show the equation of state, i.e., the energy density and pressure as functions of temperature. In the next section we will show that the ratios of the standard transport coefficients to their corresponding relaxation times is governed by the equation of state. This has a consequence that the theory that gives the equation of state should also be able to yield the transport coefficients if one wishes to calculate the two self-consistently.



**Figure 1.** The equation of state: energy density and pressure as functions of temperature for pure pionic matter.

### 3. Transport coefficients of hot and dense nuclear matter

The transport coefficients are generally calculated using the relativistic kinetic theory and thereby imply the knowledge of a collision term. The relaxation times are considered to be given by more sophisticated theory or evaluated roughly, so that we compare the standard transport coefficients (thermal conductivity  $\kappa$ , bulk viscosity  $\zeta$  and shear viscosity  $\eta$ ) divided by their associated relaxation times (these are the relaxation times for the heat flux  $\tau_q$ , for the bulk viscous pressure  $\tau_\Pi$  and for the shear viscous pressure  $\tau_\pi$ ). For the thermal conductivity we will use  $\lambda = \kappa T$  which is the quantity that enters the dissipative fluid dynamic equations [5]. Thus we will consider the ratios  $\lambda/\tau_q$ ,  $\zeta/\tau_\Pi$  and  $\eta/\tau_\pi$ . For the sake of brevity, only the calculations performed for pure pionic matter are presented in the results. From Ref. [9] the ratios of transport coefficients to their associated relaxation times are related to the  $\beta_A$  by

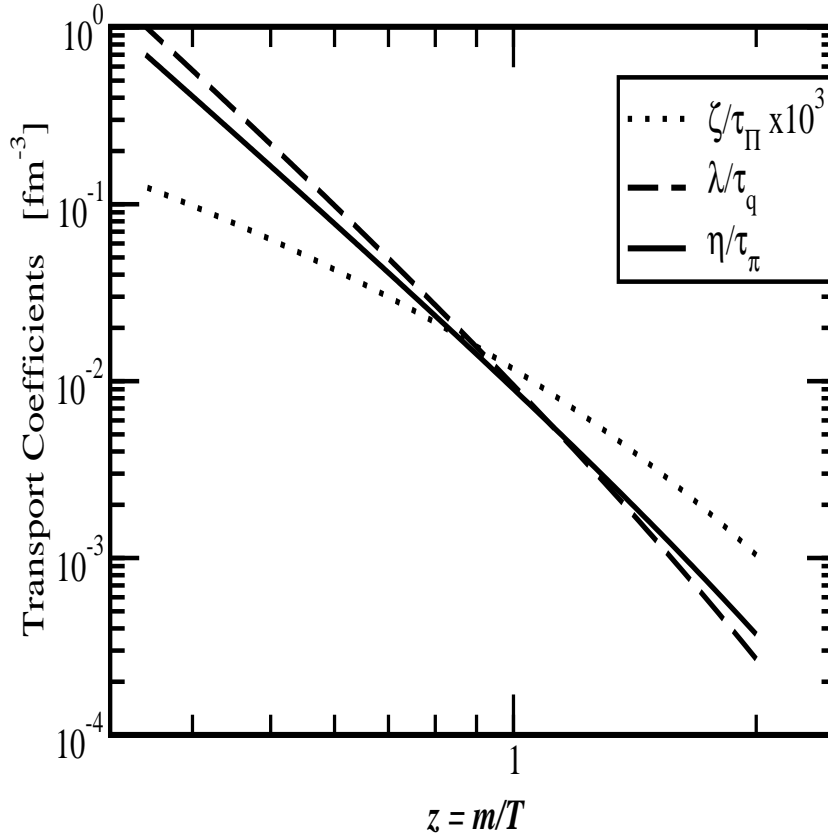
$$\frac{\zeta}{\tau_\Pi} = \frac{1}{\beta_0}, \quad (4)$$

$$\frac{\lambda}{\tau_q} = \frac{\kappa T}{\tau_q} = \frac{1}{\beta_1}, \quad (5)$$

$$\frac{2\eta}{\tau_\pi} = \frac{1}{\beta_2}, \quad (6)$$

The second order coefficients, i.e.,  $\beta_0(\varepsilon, n)$ ,  $\beta_1(\varepsilon, n)$  and  $\beta_2(\varepsilon, n)$  are given by the equation of state. They are all positive and they make the second order theories of relativistic fluid dynamics causal [5]. They are presented in detail in Ref. [9]. Thus the ratios of the transport coefficients to relaxation times should be governed by the equation of state.

In Fig. 2 we show the transport coefficients divided by their respective relaxation times, i.e.,  $\lambda/\tau_q$ ,  $\zeta/\tau_\Pi$  and  $\eta/\tau_\pi$ .



**Figure 2.** Transport coefficients  $\zeta/\tau_\Pi$ ,  $\lambda/\tau_q$ ,  $\eta/\tau_\pi$ , as functions of the "temperature parameter"  $z = m/T$  for pure pionic matter.

The bulk viscosity of the pionic matter is very small. In Fig. 2 it is magnified one thousand times in order to compare its temperature dependence with the other two coefficients. The thermal conductivity times temperature, i.e.,  $\lambda$  is comparable to the shear viscosity. Since the transport coefficients of the pionic matter increases with temperature and the relaxation times decreases with temperature [8, 10] one would expect the ratio of these transport coefficients to their associated relaxation times to increase with rising temperature. The  $\beta_A$  are inversely proportional to the pressure. The ratios of the transport coefficients to their associated relaxation times are inversely

proportional to the  $\beta_A$  and thus directly proportional to the pressure. Hence like the pressure the ratios of the transport coefficients to their associated relaxation times increases with temperature.

#### 4. Conclusions and outlook

A specific model has to be chosen to describe the space-time evolution of relativistic nuclear matter. Using relativistic dissipative fluid dynamics necessitates the knowledge of the underlying equation of state and the transport coefficients or relaxation times for dissipative fluxes. It has been shown that the model/theory that gives the equation of state of the system under consideration should be the same that gives the transport coefficients of the system for consistency. In our calculations we considered a  $\pi\pi$  system. It is clear from this simple study that the transport coefficients are as important as the equation of state.

In this work we focused on the three standard transport coefficients, namely the thermal conductivity, the bulk viscosity and the shear viscosity. This is relevant in the present case study of a single component system: the  $\pi\pi$  system. When one studies a multi-component mixtures the diffusion coefficients become important, e.g., flavor diffusion. In the case of flavor diffusion the particle flavors are initially separated spatially. That is, the flavor chemical potential depends on position. The flavor will be flowing with an associated flow velocity. That is, we can define a drift velocity for each component of the mixture independently of the four-flow velocity as a whole. For example in a pion-kaon  $\pi K$  mixture the kaons will flow with a certain velocity and they will drift through the pions. In the case of a mixture of more than two components the calculations become complex and one might resort to microscopic transport models [10, 11].

- [1] A. Muronga, Phys. Rev. C **69**, (2004) 044901.
- [2] A. Muronga and D.H. Rischke nucl-th/0407114.
- [3] U. W. Heinz, H. Song and A. K. Chaudhuri, Phys. Rev. C **73**, (2006) 034904
- [4] R. Baier, P. Romatschke and U. A. Wiedemann, Phys. Rev. C **73**, (2006) 064903.
- [5] A. Muronga, Phys. Rev. C **76**, (2007) 014909.
- [6] T. Koide, G.S. Denicol, Ph. Mota and T. Kodama, hep-ph/0609117.
- [7] H. Heiselberg, G. Baym and C.J. Pethick, in *Strange Quark Matter in Physics and Astrophysics*, edited by J. Madsen and P. Haensel [Nucl. Phys. B (Proc. Suppl.) **24B**, 144(1991)]
- [8] M. Prakash, M. Prakash, R. Venugopalan and G. Welke, Phys. Rep. **227**, (1993) 321.
- [9] A. Muronga, Phys. Rev. C **76**, (2007) 014910.
- [10] A. Muronga, Phys. Rev. C **69**, (2004) 034903.
- [11] N. Sasaki, O. Miyamura, S. Muroya and C. Nonaka, Phys. Rev. C **62**, (2000) 011901R; N. Sasaki, O. Miyamura, S. Muroya, C. Nonaka, Europhys. Lett. **54** (2001) 38.